

# 1 Differentialgleichungen

## 1.1 Separation der Variablen

$$\frac{d}{dt}x(t) = f(x(t))g(t), \quad x(t_0) = x_0$$

$$\int_{x_0}^{x(t)} \frac{dy}{f(y)} = \int_{t_0}^t g(s) ds$$

## 1.2 Lineare Differentialgleichungen 1. Ordnung

$$\dot{x}(t) = cx(t) + g(t)$$

$$x(t) = x(t_0)e^{c(t-t_0)} + \int_{t_0}^t e^{c(t-s)}g(s) ds$$

## 1.3 Lineare hom. Differentialgleichungen 2. Ordnung

$$\ddot{x}(t) = c_1\dot{x}(t) + c_0x(t) \Rightarrow \ddot{y} + py + qy = 0$$

$$\lambda^2 + p\lambda + q = 0$$

$$\lambda_{1,2} = -\frac{p}{2} \pm \frac{1}{2}\sqrt{p^2 - 4q}$$

$$p^2 > 4q \Rightarrow y_h(t) = C_1e^{\lambda_1 t} + C_2e^{\lambda_2 t}$$

$$p^2 = 4q \Rightarrow y_h(t) = C_1e^{\lambda t} + C_2te^{\lambda t}, \quad \lambda = -\frac{p}{2}$$

$$p^2 < 4q \Rightarrow y_h(t) = e^{-\frac{pt}{2}}(C_1 \cos(\omega t) + C_2 \sin(\omega t)), \quad \omega = \frac{1}{2}\sqrt{4q - p^2}$$

## 1.4 Lineare inhom. Differentialgl. 2. Ordnung

$$\ddot{y} = c_1\dot{y} + c_0y + g$$

$$y = y_h + y_i, \quad \ddot{y}_h = c_1\dot{y}_h + c_0y_h$$

$$1: y_1\dot{c}_1 + y_2\dot{c}_2 = 0, \quad 2: \dot{y}_1\dot{c}_1 + \dot{y}_2\dot{c}_2 = 0$$

## 1.5 Nichtlineare Differentialgleichungen

$$\dot{y} = p q(y)$$

$$\frac{dy}{q(y)} = p dt \Rightarrow \int \frac{dy}{q(y)} = \int p dt + C$$